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EXPERIENCE OF OPTIMIZATION OF AERODYNAMIC CHARACTERISTICS OF AIRFOILS

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The complex approach developed previously for the numerical solution of problems of optimization and designing of airfoils is applied to solve the problem of increasing the lift-to-drag ratio of subsonic airfoils used.

Introduction. One of the methods of improving flying vehicles is the design of airfoils that possess necessary properties under given restrictions. The airfoil should have the optimal characteristics in cruising flight. Such an airfoil can be simulated at the current stage of development of numerical methods and computational equipment [1–5]. Nonformalized design methods allow one to construct airfoils that satisfy initial parameters (the airfoils obtained belong to the region of parameters admissible by geometric and gas-dynamic restrictions) but are not optimal, since the principle of optimization is not inherent in the design technology [6]. Mathematical methods of solving optimization problems allow one to improve the characteristics of the airfoils used and obtain new solutions in a prescribed allowed set.

A direct method of optimization has been actively developed lately. It is based on using numerical methods of optimization and computational methods of fluid dynamics. In this case, *a priori* information on the solution is not needed. The main difficulty is the large computer time spent on computation and analysis of the behavior of derivatives of the objective function with respect to design variables. Under these conditions, the choice of the gas-flow model, the numerical method for solving the corresponding equations, and the method of optimization is very important. Most papers dealing with design and optimization of airfoils employ either the Euler model or the model of viscid–inviscid interaction as the gas-flow model. The use of the Navier–Stokes equations in solving optimization problems requires the use of supercomputers and algorithms for computation parallelism. At the same time, the question on the adequacy of turbulence models used under conditions of arbitrary variation of design variables remains open. The use of the viscid–inviscid interaction model for solving the problem of the flow around an airfoil is more rational from the viewpoint of both obtaining practically relevant results of solving optimization problems and computational efficiency. Within the framework of this approach, an algorithm was developed for the numerical solution of boundary-value problems of the subsonic gas flow around airfoils on the basis of the method of boundary elements for solving a nonlinear integral equation on adaptive grids, which allows a successful solution of direct optimization problems. Since the flow around airfoils occurs at high Reynolds numbers, the calculation is performed within the framework of integral relations of the boundary-layer model. Such an approximation is justified in this case (which is evidenced by the results of comparison of calculated and experimental polars) and allows one to avoid repeated solutions of boundary-layer equations under conditions of possible separation upon variation of the contour shape.

1. Formulation of the Problem. The NACA airfoil 64_2-215 was chosen to test the calculation method developed [7]. The calculated and experimental characteristics are in good agreement (Figs. 1 and 2). Figure 1 shows the distribution of the pressure coefficient c_p over the upper and lower surfaces of the airfoil (the value of the x coordinate is normalized to the airfoil chord length b_c and c_p^* is the critical value of the pressure coefficient). Figure 2 shows the polar for this airfoil (c_x is the drag coefficient and c_y is the lift coefficient). It is essential

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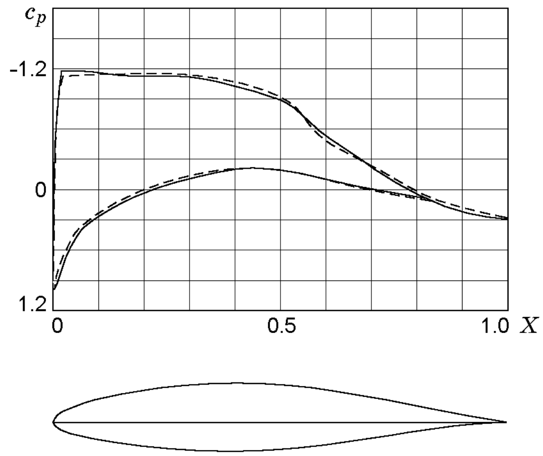

Fig. 1

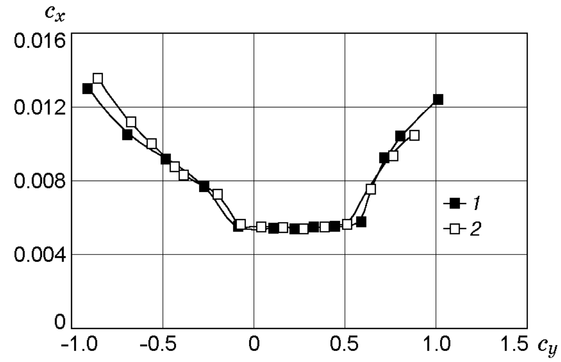

Fig. 2

Fig. 1. Distribution of the pressure coefficient over the upper and lower surfaces of the NACA airfoil 64_2-215 ($c_p^* = -1.294$, M = 0.6, and angle of attack $\alpha = 4°$): the solid and dashed curves refer to the calculation and experiment, respectively.

Fig. 2. Experimental (1) and calculated (2) polars of the NACA airfoil 64_2-215.

that the NACA airfoil 64_2-215 has high aerodynamic characteristics and is, apparently, close to the optimal shape. Therefore, it can serve as a representative object for verification of the optimization problem and the method of its solution. The results obtained in optimization are also of practical interest.

The optimization problem is formulated as follows. We have to construct an airfoil that satisfies the gas-dynamic constraints

$$\mathrm{M}_\infty = \text{const}, \qquad \mathrm{M}_{c\,\max} \leqslant \mathrm{M}_{\max}, \qquad \mathrm{Re} = \mathrm{Re}_0, \qquad f(s) \geqslant f_0,$$

and the geometric constraints

$$d_{\min} \leqslant d_{c\,\min}, \quad d_{c\,\max} \leqslant d_{\max}, \quad S_{\min} \leqslant S_c, \quad b_c = \text{const}, \quad F(x, \boldsymbol{p}) \in C^k(0, b_c),\ k \geqslant 1$$

and ensures the maximum (minimum) of the objective functional $\Phi(\boldsymbol{p}, \boldsymbol{g})$:

$$\Phi(\boldsymbol{p}, \boldsymbol{g}) = K_a, \qquad \boldsymbol{g} = (\mathrm{M}_\infty, \mathrm{M}_{\max}, \mathrm{Re}_0, f_0, d_{\min}, d_{\max}, S_{\min}).$$

Here M_∞ is the free-stream Mach number, $\mathrm{M}_{c\,\max}$ is the maximum local Mach number on the contour, $\mathrm{M}_{\max}$ is a prescribed maximum value of the Mach number, Re is the Reynolds number, $d_{c\,\min}$ and $d_{c\,\max}$ are the minimum and maximum thicknesses of the airfoil in percentage of the chord length, S_c is the airfoil area, f_0 is a constant that enters the chosen detachment criterion, s is the length of the arc along the upper and lower contours of the airfoil, $d_{\min}$ and $d_{\max}$ are the allowable minimum and maximum thicknesses of the airfoil, $S_{\min}$ is the allowable minimum area covered by the airfoil contour, $F(x, \boldsymbol{p})$ is a function that describes the contour configuration and depends on the vector of parameters $\boldsymbol{p}$, k is the degree of contour smoothness, and K_a is the lift-to-drag ratio. In the problem considered, we have $\boldsymbol{g} = (0.5, 1, 10^6, -3, 0, 15\%)$, and no constrains on the area were imposed.

The problem considered reduces to the problem of nonlinear programming, which is solved in our case by a gradient-free method of search with adaptation and the use of an element of randomness. Different strategies of obtaining the optimal solution are considered. A special feature of determining the optimal contour of the airfoil is the choice of the method for including the angle of attack into the set of parameters varied in calculating the functional. In this case, one of two approaches can be used: 1) the angle of attack is included into the set of varied parameters; because of the strong effect of variation of the angle of attack on variation of the functional, the functional-level surfaces have a ravine structure, and it is difficult to reach the extreme point; 2) the angle of attack at which the extreme value of the functional is reached is determined at each variation of parameters. If the constraints include the condition of equality of the lift force to a given value, the angle of attack corresponding to a given restriction for each variation of geometric parameters is found in the second approach. The second strategy allows one to obtain the best (i.e., maximum or minimum) value of the objective functional.

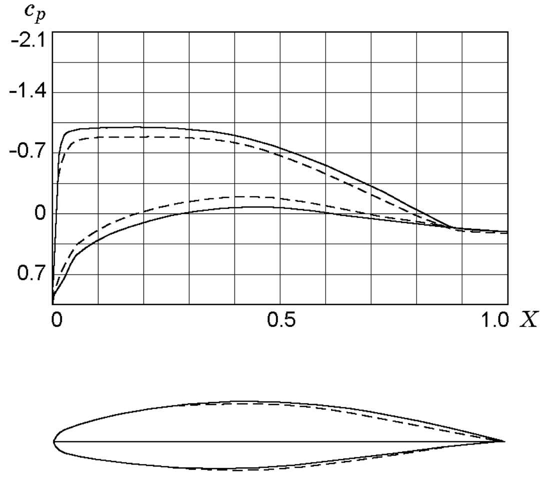

Fig. 3

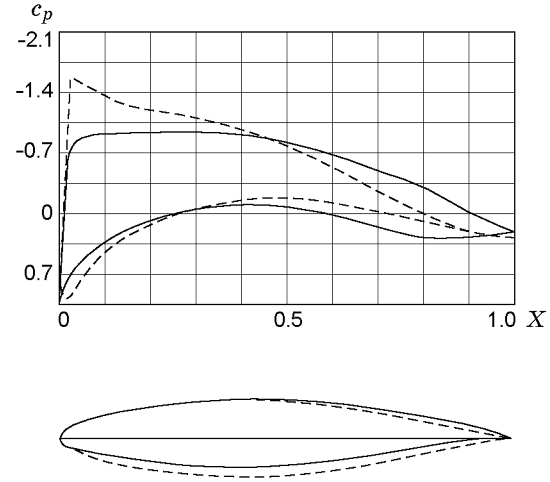

Fig. 4

Fig. 3. Initial NACA airfoil 64_2-215 (dashed curve), optimal airfoil (solid curve), and the corresponding distributions of the pressure coefficient.

Fig. 4. Initial symmetric airfoil (dashed curve), optimal airfoil (solid curve), and the corresponding distributions of the pressure coefficient.

TABLE 1

Calculation Results without Constraints on c_y

Airfoil number	c_y	c_x	c_m	K_a
1	0.551	0.0096	−0.037	57.4
2	0.718	0.0122	−0.063	59.0
3	0.801	0.0125	−0.136	64.1

TABLE 2

Calculation Results for a Given Value of c_y

Airfoil number	c_y	c_x	c_m	K_a
1	0.605	0.0117	−0.037	51.8
2	0.602	0.0114	−0.062	53.0
3	0.602	0.0101	−0.125	59.7

2. Calculation Results. Figures 3 and 4 show the results of solving the problem of maximization of the lift-to-drag ratio, the initial contours being the NACA airfoil 64_2-215 and symmetric airfoil, respectively. The initial and final distributions of the pressure coefficient and airfoil contours are plotted by dashed and solid curves, respectively. The angle of attack corresponds to the maximum value of the lift-to-drag ratio. In Figs. 3 and 4, we have $X = x/b_c$.

The integral aerodynamic characteristics of the initial airfoil and optimal airfoils obtained are listed in Tables 1 and 2 (No. 1 is the initial NACA airfoil 64_2-215, No. 2 is the optimal airfoil obtained from airfoil 1, and No. 3 is the optimal airfoil obtained from the symmetric airfoil). The lift-to-drag ratio of airfoil 2 was improved mainly by increasing the curvature of the upper contour and shifting it toward the rear part. This allowed an increase in the lift force with an insignificant increase in drag of the airfoil. The airfoil obtained from the symmetric initial contour has an even higher lift-to-drag ratio caused by much greater "trimming" of the rear part of the lower contour, which allowed an increase in the lift force with a simultaneous decrease in the angle of attack. This, in turn, led to a "plateau" in the velocity distribution on the upper surface of the contour. An extended section of the laminar boundary layer was formed thereby, which reduced the increase in drag inevitable when adverse pressure gradients appear in the rear section with "trimming." It follows from Table 1 that the absolute value of the coefficient c_m in the rear part of the airfoil increases significantly with increasing aerodynamic load. In addition, for the lift coefficient $c_y = 0.6$ in the cruising regime, the lift-to-drag ratio of the airfoils obtained is greater than that of the initial airfoil (see Table 2). The reason is the decrease in drag of the airfoils obtained for a fixed value of the lift coefficient. If the maximum airfoil thickness remains unchanged, the results of solving the problems in an identical formulation are different for different initial contours. For the NACA airfoil 64_2-215, this constraint is satisfied from the very beginning. The thickness of the initial symmetric airfoil is greater than the prescribed value. The solution of the optimization problem reduces to minimization of the composite functional including the objective and "penalty" functionals. The structure of the "penalty" functional is such that it increases monotonically if the constraints are violated. Therefore, the descent trajectory upon minimization of the composite functional depends on the position of the initial point in the space of parameters that define the geometry. Since the composite functional is not convex, various local extremes appear.

The method developed does not allow one to calculate aerodynamic characteristics in separated flows, in particular, the maximum lift coefficient $c_{y\,\max}$. Nevertheless, a comparison of the dependences of the shape factor and lift coefficient on the angle of attack for the initial airfoil and resultant optimal airfoils allow us to conclude that the value of $c_{y\,\max}$ does not decrease for the optimal configurations. The final conclusion about the advantages of the airfoils designed over other airfoils constructed with regard for the constraints formulated can be made only after experimental studies. Nevertheless, the validity and reliability of the results obtained, which are based on the use of the known models of fluid mechanics in the wing theory, careful testing of methods developed and used for solving the equations of motion and optimization problems, correctness of their formulation, and agreement of the results obtained with available experimental data and exact solutions, allow one to reduce the scale of these studies and obtain new results inaccessible by means of parametric search for standard geometric parameters of the airfoil.

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